

# TESTING AXIOMATIZATIONS OF AMBIGUITY AVERSION

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**Abstract** The study of the normative and positive theory of choice under uncertainty has made major advances through thought experiments often referred to as paradoxes: the St. Petersburg paradox, the Allais paradox, the Ellsberg paradox, and the Rabin paradox. Machina proposes a new thought experiment which posits a choice between two acts that have three outcomes. As in the Ellsberg paradox there are three events, but while the Ellsberg paradox has two (monetary) outcomes in Machina there are three. Machina shows that four prominent theories of ambiguity aversion predict indifference between the acts. Introspection, however, suggests that many people might very well strictly prefer one act over the other. This paper makes four contributions: first, to our knowledge, it is the first to experimentally implement the Machina thought experiment. Second, we employ a novel method to simultaneously elicit the certainty equivalent of an embedded lottery. Third, our results—across three experiments—indicate non-indifference, which rejects earlier theories of ambiguity aversion, but is consistent with a newer one, which we apply to explain our results. Fourth, we show that independence is a sufficient condition for indifference in the Machina paradox, and thereby explains why so many models predict indifference.

**JEL Codes:** D81

**Keywords:** Ellsberg paradox, Machina paradox, uncertainty aversion, independence axiom

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## 1 Introduction

The development of the normative and positive theory of behavior under uncertainty is characterized by a series of thought experiments to which scholars or laypersons often give a “wrong” answer. This is why these thought experiments are often referred to as paradoxes. The first thought experiment, the St.-Petersburg-Paradox, was proposed by Nicolas Bernoulli (see de Montmort, 1713) and challenged the notion that a lottery will be evaluated by its expected value. Daniel Bernoulli (1738) suggested a theory to accommodate observed behavior by using a concave utility function instead of the payoffs themselves. Centuries later that theory of using a real-valued function of the outcomes was put on normative foundations by von Neumann and Morgenstern (1944). Allais (1953) then challenged that theory, proposing a thought experiment demonstrating that many people do not exhibit the behavior suggested by Bernoulli and von Neumann and Morgenstern.

Expected utility theory concerns situations where a probability distribution is exogenously given. Savage (1954) proposed to expand this theory to situations where no probability distribution is given, and formulated axioms that imply that a decision-maker would have a single subjective probability distribution and follow expected utility theory. Ellsberg (1961) proposed a thought experiment that challenges the notion that decision-makers have a single subjective probability distribution (i.e., are probabilistically sophisticated). Empirical papers followed (for a survey see Camerer and Weber, 1992), showing that there is a “paradox,” (i.e., that people behave differently than probabilistic sophistication prescribes). New models were proposed to accommodate the behavior observed in the Ellsberg experiment (e.g., exhibiting ambiguity non-neutrality).

These new models were designed to accommodate ambiguity aversion, or behavior that might be seen as Ellsberg-paradoxical. Thus, no conceivably observable behavior in the Ellsberg experiment can falsify these new models. Machina (2014) lists four major such models: Schmeidler’s (1989) Choquet model (or Rank-Dependent Utility); Gilboa and Schmeidler’s (1989) maximin expected utility; the smooth ambiguity model by Klibanoff et al. (2005); and the Variational Preferences Model by Maccheroni et al. (2006). Machina’s thought experiment is a test of these four theories: Machina proposes two acts, and these four models all predict indifference between the two Machina acts. Models which might not necessarily predict indifference are Segal’s (1987) recursive ambiguity model in combination with Gul’s (1991) disappointment aversion<sup>1</sup>, and Gul and Pesendorfer’s (2010) Expected Uncertain Utility Theory.

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<sup>1</sup>See Dillenberger and Segal (2015)

In the classic Ellsberg urns, there are two, three, or four states, but never more than two outcomes. Machina (2014) proposes acts with three outcomes, and proceeds to show that four major theories of ambiguity predict indifference between the two acts he constructs. An urn contains 3 balls, exactly 1 of which is red, while the other two could be both white, both black, or one white and one black ball. The outcomes in this Machina thought experiment are monetary prizes of \$0, \$c and \$100, where  $\$c \sim (\frac{1}{2}, \$0; \frac{1}{2}, \$100)$ , the certainty equivalent of the lottery of receiving \$100 with probability 50% and else \$0.

Act 1			Act 2		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
\$0	\$c	\$100	\$0	\$c	\$100

Figure 1: Machina experiment

According to Machina, “If ambiguity aversion somehow involves ‘pessimism,’ mightn’t an ambiguity averter have a strict preference for [Act] II over [Act] I, just as a risk averter might prefer bearing risk about higher rather than lower outcome levels?”

In this paper, we carefully implement the Machina test to see if people violate indifference, and if so, whether this is an economically substantial phenomenon. We find that people do violate indifference—systematically across three experiments for over 700 subjects—but violate indifference in the opposite manner than what Machina hypothesized. To the best of our knowledge, this paper is the first one to experimentally employ the Machina thought experiment.<sup>2</sup> It also discusses the Machina thought experiment and shows that for decision-makers who satisfy independence (we make precise which independence axiom we mean), the Machina thought experiment is problematic. Finally, we explain the direction of the violation of indifference using Dillenberger and Segal (2015).

To preview the theoretical observation regarding independence, replace \$c with the lottery it is induced by, so the original Machina choice becomes:

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<sup>2</sup>Machina also proposed earlier thought experiments in Machina (2009). Machina distinguishes his 2014 thought experiment, which is based on a single source of purely subjective uncertainty, unlike Machina (2009), which is based on two. Baillon et al. (2011) and L’Haridon and Placido (2010) theoretically and empirically investigated Machina’s earlier thought experiment, though the empirical tests did not control for indifference. Their results complement ours, and together, advance the argument that Machina’s paradoxes falsify many ambiguity theories, at least in the Anscombe-Aumann framework adopted by those theories with the independence axiom as central.

Act 1			Act 2		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
$\frac{1}{2}$ <b>\$0</b>	$\frac{1}{2}$ <b>\$0</b>	$\frac{1}{2}$ <i>\$100</i>	$\frac{1}{2}$ <b>\$0</b>	$\frac{1}{2}$ \$0	$\frac{1}{2}$ \$100
$\frac{1}{2}$ \$0	$\frac{1}{2}$ \$100	$\frac{1}{2}$ <i>\$100</i>	$\frac{1}{2}$ <b>\$0</b>	$\frac{1}{2}$ <i>\$100</i>	$\frac{1}{2}$ <i>\$100</i>

Figure 2: Machina experiment and reduction

Note that \$0 occurs with one-third probability and \$100 occurs with one-third probability. That is, once we substitute the certainty equivalent  $c$  with the underlying lottery, the lotteries are now identical in their objective and subjective aspects. Thus, a problem with the Machina thought experiment is its interpretation, namely, does a finding of a strict preference show that ambiguity aversion varies in wealth, or does it show a violation of reduction? By reduction, we mean that a decision-maker is indifferent to replacing the certainty equivalent as the prize by its underlying lottery.

## 2 Machina thought experiment

In Machina's thought experiment, major theories of ambiguity aversion predict indifference, so the thought experiment is posed as a test of these theories. We make two observations. First, probabilistically sophisticated non-Expected Utility (non-EU) decision makers (DM) can fail to be indifferent. We present an example (disappointment aversion) where decision makers have a strict preference. Second, any non-probabilistically sophisticated Expected Utility decision maker is indifferent. We show that for any prior, someone who satisfies the independence axiom will be indifferent. Machina's thought experiment appears at least as much a test of independence as of ambiguity aversion.

**2.1 Example of probabilistically sophisticated DM with  $Act I \approx Act II$**  Let the probabilistic sophisticated DM have:  $p_B = \frac{2}{3}, p_W = 0$ . Then, suppose the DM has non-EU Gul's (1991) disappointment aversion ( $\beta > 0$ ). Then, for any lottery with 2 outcomes  $\underline{x} < \bar{x}$  Gul's functional is simply:  $v(lottery) = \frac{(1+\beta)p(\underline{x})u(\underline{x})+p(\bar{x})u(\bar{x})}{1+\beta p(\underline{x})}$ . Normalize  $u(0) = 0, u(100) = 100$ . Then,  $u(c) = v(\$0; \frac{1}{2}, \$100; \frac{1}{2}) = \frac{\frac{1}{2}100}{1+\frac{1}{2}\beta} = \frac{100}{2+\beta}$ . Next,  $v(I) = \frac{(1+\beta)\frac{2}{3}u(0)+\frac{1}{3}u(100)}{1+\beta\frac{2}{3}} = \frac{100}{3+2\beta}$  and  $v(II) = \frac{(1+\beta)\frac{1}{3}u(0)+\frac{2}{3}u(c)}{1+\beta\frac{1}{3}} = \frac{2u(c)}{3+\beta} = \frac{200}{(2+\beta)(3+\beta)}$ . Thus  $v(II) < v(I) \Rightarrow Act I \succ Act II$ .

**2.2 Non-probabilistically sophisticated EU DM with  $Act I \sim Act II$**  Machina proposes acts with three outcomes. First, the purely objective act is:

Act 0		
1 ball	1 ball	1 ball
Black	White	Red
\$0	\$c	\$100

Then, he proposes two acts that have ambiguity either at the lower two outcomes or at the higher two outcomes:

Act L			Act H		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
\$0	\$c	\$100	\$0	\$c	\$100

Now consider 2 acts that are constructed by replacing the certainty equivalent in the Machina Acts with the underlying lottery. Note that the acts have an identical mapping from states to outcomes. This inspires our later claim that the Anscombe-Aumann axiom of Substitution together with Ordering (completeness and transitivity) and the classical independence axiom from expected utility theory are sufficient to imply indifference between Machina's acts  $L$  and  $H$ .

Act L'			Act H'		
2 balls		1 ball	1 ball	2 balls	
black	white	red	red	black	white
$\frac{1}{2}$ <span style="color: green;">\$0</span>	$\frac{1}{2}$ <span style="color: green;">\$0</span>	$\frac{1}{2}$ <span style="color: blue;">\$100</span>	$\frac{1}{2}$ <span style="color: green;">\$0</span>	$\frac{1}{2}$ <span style="color: blue;">\$0</span>	$\frac{1}{2}$ <span style="color: blue;">\$100</span>
$\frac{1}{2}$ <span style="color: green;">\$0</span>	$\frac{1}{2}$ <span style="color: green;">\$100</span>	$\frac{1}{2}$ <span style="color: blue;">\$100</span>	$\frac{1}{2}$ <span style="color: green;">\$0</span>	$\frac{1}{2}$ <span style="color: blue;">\$100</span>	$\frac{1}{2}$ <span style="color: blue;">\$100</span>

**2.3 The Machina Acts in the Anscombe-Aumann Framework** The Machina acts have both subjective events and objective ones, which is why we can represent them in the framework by Anscombe and Aumann (1963). We follow the exposition by Machina and Schmeidler (1995):

$\mathcal{X} = \{\dots, x, \dots\}$  set of outcomes (e.g., money)

$\mathcal{S} = \{\dots, s, \dots\}$  set of states

$R = (x_1, p_1; \dots; x_m, p_m)$  a roulette lottery (purely objective)

$H = [x_1 \text{ on } E_1; \dots; x_n \text{ on } E_n]$  horse race (purely subjective)

$H^R = [R_1 \text{ on } E_1; \dots; R_n \text{ on } E_n]$  horse race (purely subjective)

$\mathcal{L} = \{\dots, L, \dots\}$  the combined set of all pure roulette, pure horse, and horse/roulette lotteries

Thus in our context we have:

The set of outcomes is  $X = \{0, c, 100\}$ .

The prize  $c$  is implicitly defined by  $c \sim (\frac{1}{2}; 0, \frac{1}{2}; 100)$ .

**2.4 State space: balls in urn** Machina uses as the state space which balls are in the urn, thus  $S = \{BB, BW, WB, WW\}$ .

Act 0 (purely objective):

$[(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$  on all states]

Act L (ambiguity at low outcomes):

$[(\frac{2}{3}; 0, \frac{1}{3}; 100)$  on  $BB$ ;  $(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$  on  $BW, WB$ ;  $(\frac{2}{3}; c, \frac{1}{3}; 100)$  on  $WW$ ]

Act H (ambiguity at high outcomes)

$[(\frac{1}{3}; 0, \frac{2}{3}; c)$  on  $BB$ ;  $(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$  on  $BW, WB$ ;  $(\frac{1}{3}; 0, \frac{2}{3}; 100)$  on  $WW$ ]

Act L' = Act H':

$[(\frac{2}{3}; 0, \frac{1}{3}; 100)$  on  $BB$ ;  $(\frac{1}{2}; 0, \frac{1}{2}; 100)$  on  $BW, WB$ ;  $(\frac{1}{3}; 0, \frac{2}{3}; 100)$  on  $WW$ ]

**2.5 State space: ball drawn** Instead of using as the state space which balls are in the urn, it might be more natural to think of the state as the ball drawn. Here the difficulty is that the ball drawn mixes objective and subjective events. Thus, we can think of the subjective state space as which ball is drawn conditional on that ball not being red, that is, have  $S = \{B, W\}$ . Another way of thinking about this is that as the red ball is taken out of the urn, one ball is drawn from the urn (horse race), and then a roulette wheel is spun where one third of the fields are red, whereas the rest of the fields have no color but, say, look at the color of the ball drawn from the urn. This approach has the advantage of yielding far shorter expressions, as it has 2 states instead of 4.

Act 0 (purely objective):

$[(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$  on all states].

Act L (ambiguity at low outcomes):

$[(\frac{2}{3}; 0, \frac{1}{3}; 100)$  on  $B$ ;  $(\frac{2}{3}; c, \frac{1}{3}; 100)$  on  $W$ ]

Act H (ambiguity at high outcomes):

$[(\frac{2}{3}; c, \frac{1}{3}; 0)$  on  $B$ ;  $(\frac{2}{3}; 100, \frac{1}{3}; 0)$  on  $W$ ]

Act L' and H':

$[(\frac{2}{3}; 0, \frac{1}{3}; 100)$  on  $B$ ;  $(\frac{1}{3}; 0, \frac{2}{3}; 100)$  on  $W$ ]

**2.6 Informational Symmetry** We assume that the DM treats the events  $B$  and  $W$  as informationally symmetric.

Ensuring or assuming information symmetry is particularly important in the context of the

Machina acts, as White yields a strictly higher prize in both acts. Informational symmetry means  $p_w = p_B$  in the ball draw state space, and  $p_{BB} = p_{WW}$  in the ball in the urn state space.

**2.7 Indifference between the Machina Acts** Under what conditions is a DM indifferent between the Machina Acts? First, observe that by informational symmetry,  $p_W = p_B$  (resp.  $p_{WW} = p_{BB}$ ), but then the DM effectively views both L and H as the lottery  $(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100)$ , and thus  $L \sim H$ . But more interestingly, what about non-probabilistically sophisticated decision-makers, when are they indifferent?

**2.8 Two kinds of independence** As Machina and Schmeidler (1995) explain, Anscombe-Aumann has four axioms, in which the first two, Ordering and Mixture Continuity are related to nonstochastic consumer theory, while the latter two, Substitution and Independence, are related to expected utility. All four together imply probabilistic sophistication (and expected utility). Focus here on three of them, abstracting from Mixture Continuity, which we do not need for present purposes.

AXIOM (Ordering)  $\succsim$  is a complete, reflexive and transitive binary relation on  $\mathcal{L}$ .

The following is what Machina and Schmeidler (1995) name the Substitution Axiom, which Anscombe and Aumann (1963) called the Monotonicity Axiom:

AXIOM (Substitution Axiom) For any pair of pure roulette lotteries  $P_i$  and  $R_i$ : If  $P_i \succ R_i$  then  $[P_1 \text{ on } E_1; \dots; P_i \text{ on } E_i; \dots; P_n \text{ on } E_n] \succ [R_1 \text{ on } E_1; \dots; R_i \text{ on } E_i; \dots; R_n \text{ on } E_n]$  for all partitions  $\{E_1, \dots, E_n\}$  and all roulette lotteries  $\{R_1, \dots, R_{i-1}, R_{i+1}, \dots, R_n\}$ .

The next axiom of Anscombe-Aumann, is an independence axiom, but they generalized it to apply to horse race/roulette lotteries, which is why we call it Horse-Race/Roulette-Independence:

AXIOM (Horse-Race/Roulette-Independence Axiom) For any partition  $\{E_1, \dots, E_n\}$  and roulette lotteries  $\{P_1, \dots, P_n\}$  and  $\{R_1, \dots, R_n\}$ :

If  $[P_1 \text{ on } E_1; \dots; P_n \text{ on } E_n] \succ [R_1 \text{ on } E_1; \dots; R_n \text{ on } E_n]$

then  $[\alpha P_1 + (1 - \alpha)Q_1 \text{ on } E_1; \dots; \alpha P_n + (1 - \alpha)Q_n \text{ on } E_n]$

$\succ [\alpha R_1 + (1 - \alpha)Q_1 \text{ on } E_1; \dots; \alpha R_n + (1 - \alpha)Q_n \text{ on } E_n]$

for all probabilities  $\alpha \in (0, 1]$  and all roulette lotteries  $\{Q_1, \dots, Q_n\}$ .

By contrast, the classical Independence Axiom (for pure roulette lotteries from expected-utility theory) is the following, and for clarity, we call it Roulette-Independence:

**AXIOM (Roulette-Independence Axiom)** *For all pure roulette-lotteries  $R, P, Q$ , and all  $\alpha \in (0, 1]$  If  $R \succ P$  then  $\alpha R + (1 - \alpha)Q \succ \alpha P + (1 - \alpha)Q$ .*

The Horse Race/Roulette-Independence Axiom implies the Roulette-Independence Axiom, while the converse is not true. Indeed the Horse Race/Roulette-Independence Axiom together with the other 3 Anscombe-Aumann axioms implies probabilistic sophistication, while Roulette-Independence does not. Many major theories of ambiguity aversion (as they are theories that allow for ambiguity non-neutrality) violate the Horse-Race/Roulette Independence Axiom, but satisfy Roulette-Independence:

**REMARK** The Multiple Priors, the Rank-Dependent Model, the Smooth Ambiguity Preferences Model, and the Variational Preferences Model satisfy Roulette-Independence.

## 2.9 Roulette-Independence: Bernoulli without Bayes?

**CLAIM** A decision-maker who satisfies the Ordering, Roulette-Independence, and Substitution Axioms is indifferent between Act L and Act H.

**PROOF:** We prove this separately in both state spaces:

1. State space: Balls in Urn:

By Roulette-Independence, we have  $(\frac{1}{3}; 0, \frac{1}{3}; c, \frac{1}{3}; 100) \sim (\frac{1}{3}; 0, \frac{2}{3}; c)$ , and  $(\frac{2}{3}; c, \frac{1}{3}; 100) \sim (\frac{1}{3}; 0, \frac{2}{3}; 100)$ . But then the Substitution Axiom implies that  $L \sim H$ , since one can substitute these lotteries on  $BB$  and  $WW$ , respectively.

2. State space: Ball Drawn:

By Roulette-Independence, we have  $(\frac{2}{3}; 0, \frac{1}{3}; 100) \sim (\frac{2}{3}; c, \frac{1}{3}; 0)$ , and  $(\frac{2}{3}; c, \frac{1}{3}; 100) \sim (\frac{1}{3}; 0, \frac{2}{3}; 100)$ . But then the Substitution Axiom implies that  $L \sim H$ , since one can substitute these lotteries on  $B$  and  $W$  respectively. *Q.E.D.*

**2.10 Indifference between subjective and objective lottery?** Note that Substitution and Roulette-Independence, unlike probabilistic sophistication, do not imply indifference between the horse-race/roulette lotteries  $L$  and  $H$  on the one hand, and the pure roulette lottery that is Act 0:

**EXAMPLE (Multiple Priors)** Let us use a simple version of the multiple priors model. Let the priors be  $p_W^1 = 0$  and  $p_W^2 = 1$ . The DM evaluates each Act by the expected utility



that nature chooses the worst prior for her. We normalize her Bernoulli utility function with  $u(0) = 0$ ,  $u(100) = 100$ , which implies  $u(c) = 50$ . Thus, the DM evaluates the acts as follows:  $V(\text{Act } 0) = \frac{1}{3}0 + \frac{1}{3}c + \frac{1}{3}100 = 50$ ,  $V(\text{Act } L) = \min \left\{ \frac{2}{3}0 + 0 \cdot c + \frac{1}{3}100, 0 \cdot 0 + \frac{2}{3} \cdot c + \frac{1}{3}100 \right\} = 33\frac{1}{3}$ ,  $V(\text{Act } H) = \min \left\{ \frac{1}{3}0 + \frac{2}{3} \cdot c + 0 \cdot 100, \frac{1}{3}0 + 0 \cdot c + \frac{2}{3} \cdot 100 \right\} = 33\frac{1}{3}$ . Thus, while the DM satisfies Roulette-Independence, she still is ambiguity averse as:  $\text{Act } 0 \succ \text{Act } L \sim \text{Act } H$ .

What about our illustrative Acts L' and H'? Under Substitution and Independence, we have:  $\text{Act } L' \sim \text{Act } L \sim \text{Act } H$ .

### 3 Experiment 1

In the first experiment, we replaced  $\$c$  with the lottery it is induced by, and asked individuals to choose an urn (lottery). For the purposes of the results discussion and continuity with the theoretical discussion, we refer to Act L' (ambiguity at low outcome) and Act H' (ambiguity at high outcome). The labeling of the urns with "A" and "B" in the instructions were chosen arbitrarily for the subjects.

A design choice was the number of balls to put in the urn. Machina parsimoniously fills his opaque urn with 1 known and 2 unknown balls. Experience shows that then some subjects assume some symmetric objective probability distribution is implied, and they mechanically start calculating the resulting distribution of this compound lottery. We avoid this by having 20 known and 40 unknown balls. This serves three purposes. First, it makes the mechanical thoughtless calculation harder. Second, it makes examples better for the experimenter, "for example, 7 black and 33 white balls". Third, Ellsberg also proposed a large number of balls.

We recruited 213 participants on Amazon Mechanical Turk and implemented a test on SurveyGizmo. Caution is warranted when interpreting these results as the instructions had several shortcomings: the labeling and order of the lotteries was not randomized, and there were minor wording issues; see the participant screen (Figure 3). Act L' was chosen by 123 participants (58%). A two-sided t-test rejects the null hypothesis that this preference for Act L' is random, at a significance level of 5% ( $p = 0.0234$ ).

5. There is an urn with 20 red balls and 40 white and black balls. The number of the white and black balls is unknown, and can be anything between 0 and 40 white balls and 0 and 40 black balls. At random, one ball will be drawn from this urn. You can pick either Lottery A or Lottery B to get paid. The payments under the two lotteries are as follows:

Lottery A: If a red ball is drawn, you will be paid 30 cents. If a black ball is drawn, you will be paid nothing. If you pick a white ball, there is a one in two chance that you will be paid 30 cents, and there is a one in two chance that will be paid 0 cents.

Lottery B: If a red ball is drawn, you will be paid 0 cents. If a black ball is drawn, there is a one in two chance that you will be paid 30 cents, and there is a one in two chance that will be paid 0 cents. If a white ball is drawn, you will be paid 30 cents.

Which one will you pick? \*

☐ Lottery A

☐ Lottery B

Figure 3

A second recruitment of 432 subjects participated in the Machina thought experiment. This time, the labeling and order of the urns (lotteries) were randomized. We used oTree (Chen et al. 2014).

## Pick a lottery

There is an urn containing 60 balls. Exactly 20 of these 60 balls are red. The remaining 40 balls are white or black. The number of the white and black balls is unknown, so there can be anything between 0 and 40 white balls and 0 and 40 black balls.

At random, one ball will be drawn from this urn. At the same time a coin is flipped. The coin can land only on head or tail. The coin is fair, that is to say symmetric. You can pick either Alternative A or Alternative B to get paid. The payments under the two alternatives are as follows:

Alternative A: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.50. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.00. If a white ball is drawn: If the coin has landed on its heads you will be paid \$0.50. If it lands on its tails, you will be paid \$0.00.

Alternative B: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.00. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.50. If a white ball is drawn: If the coin has landed on its heads you will be paid \$0.50. If it lands on its tails, you will be paid \$0.00 .

**Lottery choice\***

A

Next

## Pick a lottery

There is an urn containing 60 balls. Exactly 20 of these 60 balls are red. The remaining 40 balls are white or black. The number of the white and black balls is unknown, so there can be anything between 0 and 40 white balls and 0 and 40 black balls.

At random, one ball will be drawn from this urn. At the same time a coin is flipped. The coin can land only on head or tail. The coin is fair, that is to say symmetric. You can pick either Alternative A or Alternative B to get paid. The payments under the two alternatives are as follows:

Alternative A: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.00. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.50. If a white ball is drawn: If the coin has landed on its heads you will be paid \$0.50. If it lands on its tails, you will be paid \$0.00 .

Alternative B: If a red ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.50. If a black ball is drawn: Regardless of the result of the coin flip, you will be paid \$0.00. If a white ball is drawn: If the coin has landed on its heads you will be paid \$0.50. If it lands on its tails, you will be paid \$0.00.

**Choose an alternative:**

A

Next

Figure 4: Choice of lottery

Among these 432 subjects, 64% preferred Act L'. We also had demographic characteristics for 333 subjects. In linear probability models, Republicans were 22 percentage points more likely to prefer Act L'. Americans were 48 percentage points and Asians were 27 percentage points more likely to prefer Act H'. Marginal effects from logit and probit models were similar. We did not see significant differences in choice of ambiguity at high or low outcomes by gender (which is the focal demographic heterogeneity of a recent study on gender differences

in ambiguity aversion (Borghans et al. 2009)).<sup>3</sup>

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<sup>3</sup>Cross-cultural differences in ambiguity aversion has been less examined relative to the literature on demographic predictors of risk aversion (Weber and Hsee 1998; Von Gaudecker et al. 2011), so we do not delve into these differences further, though readers may find demographic determinants of ambiguity aversion to be of interest.

Correlates of Urn A Choice		
	(1)	(2)
	chooseA	chooseA
Mean dep. Var.	0.36	0.37
Male		0.0564 (0.0559)
Age		0.00200 (0.00249)
Republican		-0.215** (0.102)
Democrat		-0.0398 (0.0842)
American		0.475* (0.280)
Indian		0.438 (0.290)
Black		0.112 (0.120)
Hispanic		0.116 (0.116)
Native American		-0.0419 (0.173)
Asian		0.270** (0.107)
Hindu		0.0489 (0.115)
Catholic		-0.0594 (0.0934)
Religious Services		0.00468 (0.0218)
Constant	0.359*** (0.0231)	-0.260 (0.291)
N	432	333
R-sq	0.000	0.107
Standard errors in parentheses		
* p<0.10      ** p<0.05      *** p<0.01		

Figure 5: Regression analysis

On the basis of the results described thus far, subjects appear to act in the opposite manner of what Machina thought (“If ambiguity aversion somehow involves “pessimism,” mightn’t an ambiguity averter have a strict preference for [Act] II over [Act] I, just as a risk averter

might prefer bearing risk about higher rather than lower outcome levels?”). That is, across 645 individuals, on average, ambiguity at low outcomes was preferred to ambiguity at higher outcomes.

## 4 Experiment 2

**4.1 Design** The main challenge in experimental implementation of the Machina thought experiment is that one of the three monetary outcomes employs the certainty equivalent (CE) of the lottery that grants monetary prizes of 0 respectively  $\omega (= 100)$  with equal probabilities. If a decision-maker has a preference relation which satisfies continuity, then a certainty equivalent is guaranteed to exist; strict monotonicity in the monetary outcomes ensures uniqueness. However, the certainty equivalent of a subject is unknown to the experimental economist. Normatively, there are strong reasons to be risk-neutral at small stakes (Rabin 2003), but positively, past studies have shown that most people exhibit risk-aversion.

Thus, it is inappropriate to assume  $\frac{\omega}{2}$  for the certainty equivalent - that value can at most serve as an upper bound. Thus, the experimenter has to elicit the subject’s certainty equivalent prior to conducting the Machina thought experiment. This also assumes that the elicitation itself leaves the certainty equivalent unchanged, an assumption which could be violated, for example, with an income effect.

The state-of-the-art method to experimentally elicit willingness to pay for an object is still Becker et al. (1964). However, as Karni and Safra (1987) show, if that object is a lottery, then the elicitation method will yield the correct result generally only if the decision-maker satisfies the independence axiom. Note that the independence axiom can be thought of as describing behavior under objective uncertainty, and it is indeed possible to satisfy the independence axiom and to be ambiguity averse at the same time.

The BDM-mechanism, in turn, can be implemented by two methods: the mechanism itself and a simplified “list” method. In the mechanism, people are asked to state their true valuation, a price is randomly drawn, and they receive the object at the random price if their stated valuation is above it. In the “list” method, people are presented with a list of choices, each consisting of two options, the object and a valuation, and one of the indicated choices is then selected at random. From a formal point of view, the two are close cousins, the difference being that in the list method the valuation one can state is quite coarse.

Practically, however, there are bigger differences: in the list method, participants may frame each choice as separate, and not view themselves as confronting a big lottery, thus even if independence does not hold, the mechanism would work. The mechanism itself is also quite unusual for non-economists and it is far from obvious to subjects that truth-

telling is a dominant strategy. Thus, usually subjects get the opportunity to practice with the mechanism and are explicitly told that correctly stating their true valuation is optimal.

This is a problem in our setting: we want to later use the elicited value to implement the Machina paradox. Thus, it ceases to be optimal to state the true value, but rather overstating it becomes optimal. Moreover, since the probability of receiving the certainty equivalent in the Machina thought experiment is subjective, it is not possible to correct for that incentive. For these reasons, we use the PRINCE method.

**4.2 PRINCE** The PRINCE (PRior INCentive system) method is like the list method and formally equivalent to BDM (Johnson, Baillon, Bleichrodt, Li, van Dolder, and Wakker 2015). In brief, the choice question (rather than a choice option) implemented for real is randomly selected before (rather than after) the experiment. It is provided to the subjects in a tangible form (for example in a sealed envelope). Subjects’ answers are framed as instructions to the experimenter about the real choice implemented at the end. Incentive compatibility can now be crystal clear, not only to homo economicus but also to homo sapiens, and isolation is maximally salient. In the PRINCE method instead of  $\$c$ , one asks subjects for instructions for which lottery is preferred for all possible  $\$c$ .

It has the advantage over the list method in that it allows any answer, not just an answer on the list (so the valuations are not elicited coarsely). Also, the envelope is already there, and framing as “give us instructions” might lessen concerns of subjects seeing this as a big lottery when eliciting CE. To familiarize subjects with PRINCE, we first used it for a first order stochastic dominance (FOSD) task and then for CE. For the Machina experiment, we use a combination of PRINCE and the list method. Our reason for doing so is that it is a priori not clear that people have a unique switching point nor direction.

Another design issue is that the original PRINCE method tests for the endowment effect—it uses mug vs. money choice to test for endowment effect—so monotonicity in money implies that there is a threshold at which the money will be preferred. Thus, there, the instructions can take the simple form of a threshold: below  $\$y$ , I want to keep mug, above  $\$y$ , I want money. The existence of such a threshold (and even directionality) is not obvious in the Machina thought experiment. Here we ask what happens if, instead of  $\$c$ , some  $0 < x < 100$  is used?

Assume that preferences are strictly monotonic in money. Note that then there should be a certainty equivalent and it should fall between  $\$0$  and  $\$100$ . Now, consider an arbitrary  $x$  such that  $0 < x < 100$ .

Act 1			Act 2		
2 balls		1 ball	1 ball	2 balls	
Black	White	Red	Red	Black	White
\$0	\$x	\$100	\$0	\$x	\$100

Figure 6: Machina experiment

#### 4.2.1. Which preferences imply a threshold $x$ ?

A natural question is whether we can still make the argument that independence would be a sufficient condition for DM to be indifferent between the lotteries. The answer to that question is NO. An example to see why, consider the case of a small  $x$  close to 0, and a subjective expected utility (SEU) decision-maker who believes that the probability of black is zero and that of white is  $\frac{2}{3}$ . This decision maker will prefer *Act II* (which gives her \$100 with probability  $\frac{2}{3}$ ) over *Act I* (which gives her \$100 with probability  $\frac{1}{3}$ ). Consider the following simple example to show how people can switch:  $EU(I) = \frac{1}{3}100 + p_W u(x)$  and  $EU(II) = p_W 100 + p_B u(x)$ .

Example 1,  $p_W = 0$ :

$EU(I) = \frac{1}{3}100$  and  $EU(II) = \frac{2}{3}u(x)$ , thus  $Act I \succcurlyeq Act II$  iff  $x < c$

Example 2,  $p_B = 0$ :

$EU(I) = \frac{1}{3}100 + \frac{2}{3}u(x)$  and  $EU(II) = \frac{2}{3}100$ , thus  $Act II \succcurlyeq Act I$  iff  $x < c$

Example 3,  $p_W = p_B = \frac{1}{3}$ :

$EU(I) = \frac{1}{3}100 + \frac{1}{3}u(x)$  and  $EU(II) = \frac{1}{3}100 + \frac{1}{3}u(x)$ , thus  $Act I \sim Act II$  for all  $x$

#### 4.2.2. Probabilistic sophistication and SEU

Slightly more generally, since SEU implies probabilistic sophistication, we assume that  $p(White) = p_w$ , where  $p_b + p_w = \frac{2}{3}$ , and assume  $u(0) = 0$ ,  $u(\$100) = 1$ . Then  $u(c) = \frac{1}{2}$  and  $0 < u(x) < 1$ . Then  $SEU(Act I) = p_w \cdot u(x) + \frac{1}{3} \cdot 1$  and  $SEU(Act II) = p_b \cdot u(x) + p_w \cdot 1$ .

Thus, the following holds:

$$\left\{ \begin{array}{ll} \text{for } p_w > p_b : & Act I \succcurlyeq Act II \Leftrightarrow x \geq c \\ \text{for } p_w = p_b = \frac{1}{3} : & Act I \sim Act II \\ \text{for } p_w < p_b : & Act I \succcurlyeq Act II \Leftrightarrow x \leq c \end{array} \right.$$

Thus, there might be indifference, or there might be a threshold in one direction or the other.



#### 4.2.3. Probabilistic sophistication and RDU

Alternatively, under rank dependent utility (RDU), let the probability distortion/weighting function be  $f$ . Given this belief,

$$\begin{aligned} RDEU(Act I) &= f(p_w) \cdot 0 + (f(p_w + p_b) - f(p_w)) \cdot u(x) + (1 - f(p_w + p_b)) \cdot 1 \\ &= (f(p_w + b) - f(p_w)) \cdot u(x) + 1 - f(p_w + p_b) \end{aligned}$$

and

$$\begin{aligned} RDEU(Act II) &= f(\frac{1}{3}) \cdot 0 + (f(p_w + \frac{1}{3}) - f(\frac{1}{3})) \cdot u(x) + (1 - f(p_w + \frac{1}{3})) \cdot 1 \\ &= (f(p_w + \frac{1}{3}) - f(\frac{1}{3})) \cdot u(x) + 1 - f(p_w + \frac{1}{3}) \end{aligned}$$

Thus, there are three cases:

$$\left\{ \begin{array}{ll} \text{for } p_w > p_b : & Act I \succ Act II \\ \text{for } p_w = p_b = \frac{1}{3} : & Act I \sim Act II \\ \text{for } p_w < p_b : & Act II \succ Act I \end{array} \right.$$

#### 4.2.4. Non-probabilistically sophisticated beliefs/preferences

Since the event black always yields a worse outcome than the event white, in this situation the multiple priors model is behaviorally identical to a model with probabilistic sophistication and subjective probability of Black equal to  $\frac{2}{3}$ , that of White equal to 0. Thus, we are in the case of  $p_w < p_b : Act I \succcurlyeq Act II \Leftrightarrow x \leq c$ .

### 4.3 Experiment Design

#### 4.3.1. First order stochastic dominance task

The first task is the first order stochastic dominance task.

Option A		Option B	
Balls in drum	Money you get when a ball of this color is drawn	Balls in drum	Money you get when a ball of this color is drawn
4 red balls	CHF X	2 red balls	CHF X
3 white balls	CHF 9	3 white balls	CHF 9
3 black balls	CHF 7	5 black balls	CHF 7

Figure 7: Envelope content - FOSD

- ☐ If X is below or equal to CHF \_\_, \_\_ then I want **Option A**, else I will receive **Option B**.
- ☐ If X is below or equal to CHF \_\_, \_\_ then I want **Option B**, else I will receive **Option A**.

Figure 8: Answer sheet - FOSD

Note that first order stochastic dominance implies that option B is always preferred when X is less than 7.

#### 4.3.2. *Certainty equivalent task*

The second task is the CE task.

The drum is filled with 20 balls, 10 of which are white and 10 of which are black. The experimenter will draw one ball from the drum at random.

**Option A**  
If the ball is black you get CHF 20. If the ball is white you get nothing.

**Option B**  
Regardless of what color is drawn, you get CHF X.

Figure 9: Envelope content - CE

IF X is below or equal to CHF \_ \_ , \_ \_ I want Option A, otherwise I will receive Option B.

Figure 10: Answer sheet - CE

Note that someone who is risk averse would write down X less than 10.

#### 4.3.3. *Machina task*

The third task is the Machina task.

The ball will be drawn from a drum with 20 red balls and 40 balls which may be any combination of white and black balls. You do not know exactly how many white/black balls are in the drum. There are 60 balls in total in the drum. You can choose one of two options. The option payoff is dependent on the color of the ball drawn from the drum:			
	20 balls	40 balls	
	Red ball drawn	Black ball drawn	White ball drawn
Option A	CHF 0	CHF X	CHF 20
Option B	CHF 20	CHF 0	CHF X

Figure 11: Envelope content - Machina

If X is....	...I want Option A	...I want Option B	I am indifferent
CHF 0			
CHF 1			
CHF 2			
CHF 3			
CHF 4			
CHF 5			
CHF 6			
CHF 7			
CHF 8			
CHF 9			
CHF 10			
CHF 11			
CHF 12			
CHF 13			
CHF 14			
CHF 15			
CHF 16			
CHF 17			
CHF 18			
CHF 19			
CHF 20			

Figure 12: Answer sheet - Machina

Note that someone who satisfies SEU would have a unique switching point when X is CE.

**4.4 Baseline Results** We ran the experiment in Zurich. We begin our analysis with a presentation of the number of participants who fall into different categories: (i) switch from Ambiguity at Low to Ambiguity at High, (ii) switch from Ambiguity at High to Ambiguity

at Low, (iii) always choose Ambiguity at Low, (iv) always choose Ambiguity at High, (v) always indifferent, and (vi) other.

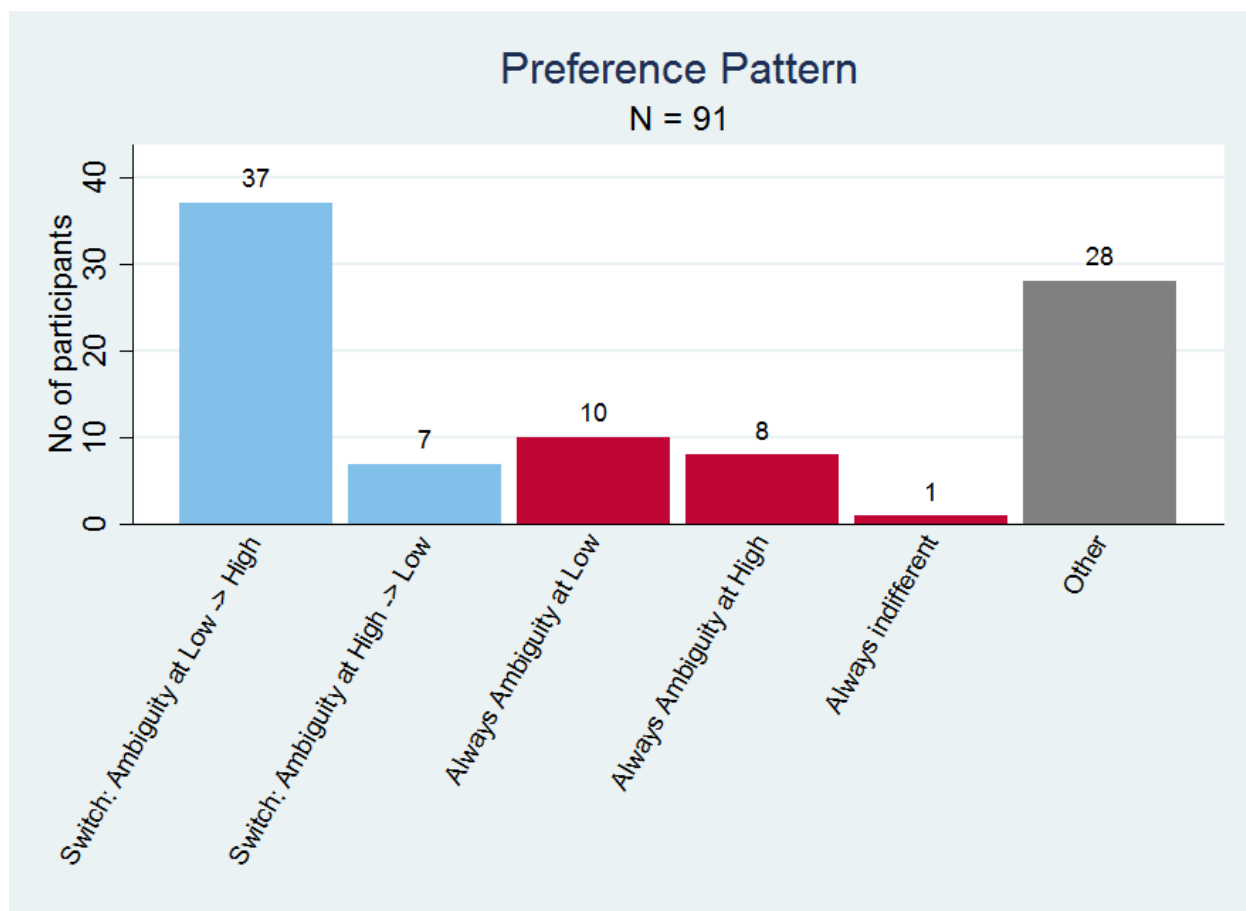


Figure 13: All participants

A few results emerge from the tabulation. First, many people do not switch. Second, there is a slight greater preference for ambiguity at low outcomes than for ambiguity at high outcomes (this echoes the findings from Experiment 1). Third, switchers switch from ambiguity at low to ambiguity at high as  $X$  increases. In the final sub-section, we present a model that explicates this result.

Next, we restrict to participants with a reasonable certainty equivalent (i.e., between 4 and 10, inclusive).

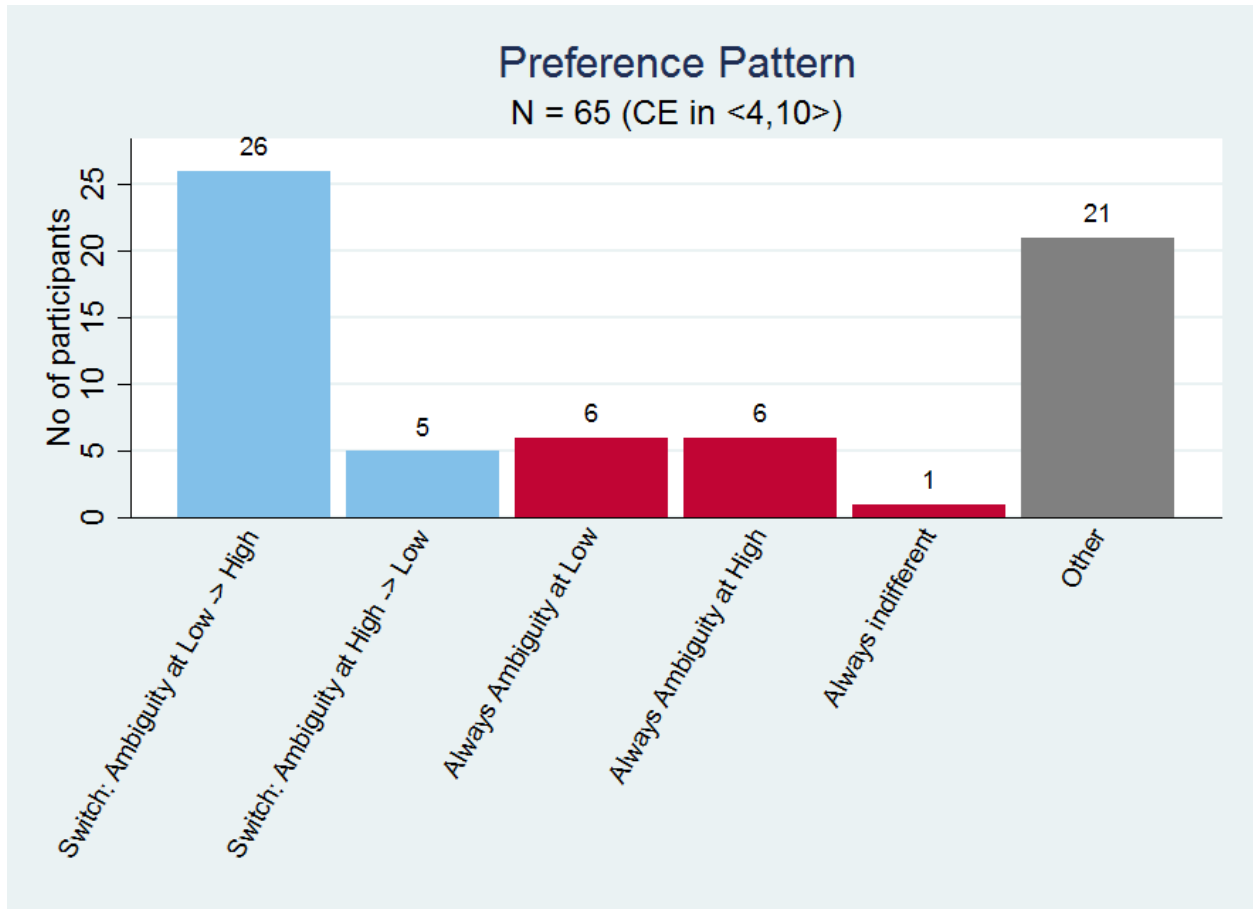


Figure 14: Participants with reasonable CE

The results are similar as without the restriction.

Next, we impose the restriction where subjects in the FOSD task chose the second option, not the first, no matter what threshold given.

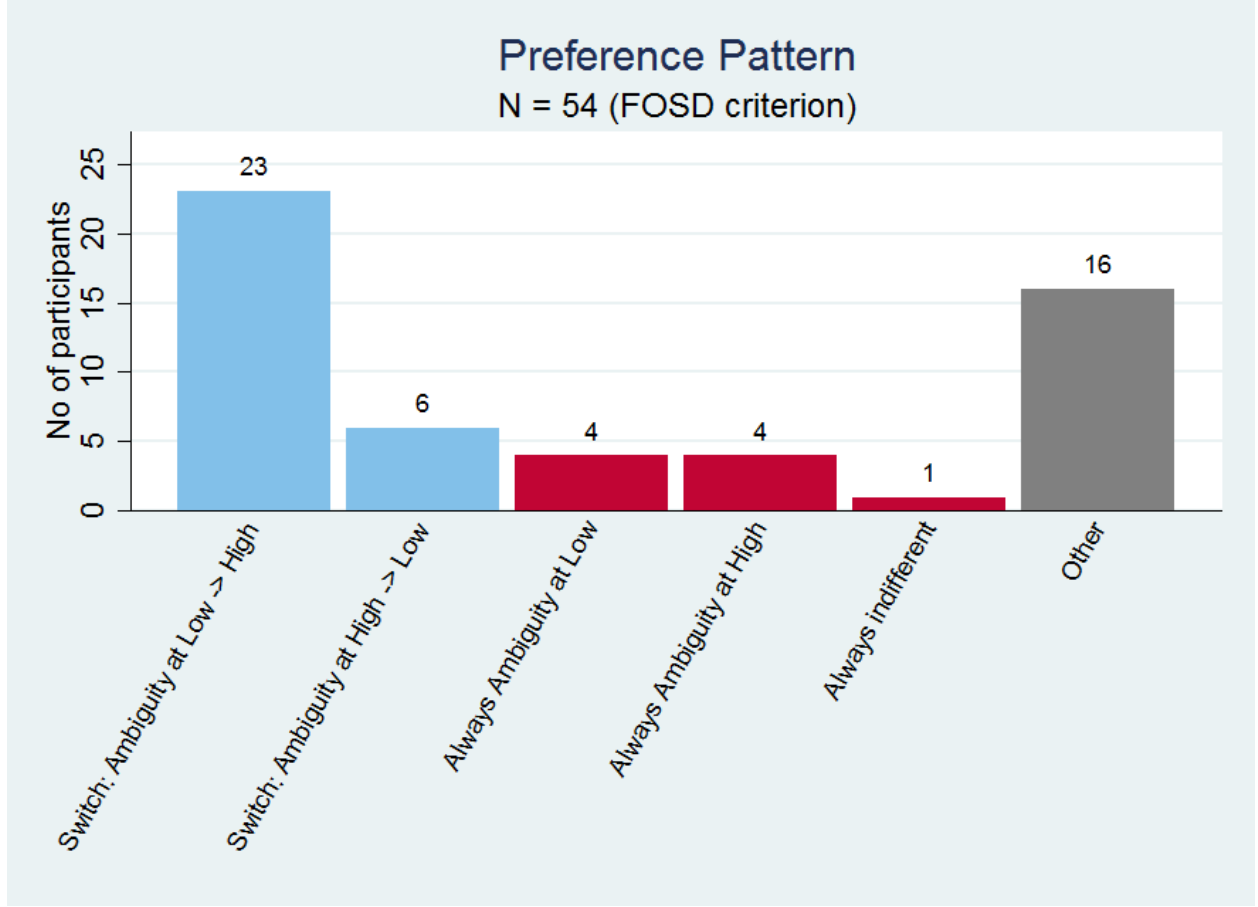


Figure 15: Participants with reasonable FOSD response

**4.5 Are people indifferent at  $X = CE$ ? No** First, we make the albeit rough approximation that indifference between *Act I* and *Act II* obtains when individuals report indifference at  $CE \pm 1$  or if  $CE \in \{S - 1.96 \cdot SD([CE - S]); S + 1.96 \cdot SD([CE - S])\}$  (where  $S$  is the switching point). More precisely,  $S$  is the average value between the last A/B and first B/A for single-switchers.  $SD$  is calculated for  $[CE - S]$ . This means that under the null hypothesis that everyone has  $CE = S$ , we treat any difference between  $CE$  and  $S$  as measurement error. In other words, subjects are classified as indifferent when they are indifferent at their  $CE$  (and two neighboring values) or they have a clear switching point and their  $CE$  lies in the confidence interval of this switching point.

In reality there are people for whom  $CE$  strongly differs from  $S$ , and thus our confidence interval is too wide. We therefore may overestimate the number of people who are indifferent.

The tabulation indicates there exists many people for whom  $CE$  strongly differs from  $S$ :

	Count	Share in %
Indifferent	49	53.8
Non-indifferent	42	46.2
Total	91	100

Figure 16: Indifference (All participants)

	Count	Share in %
Indifferent	48	71.6
Non-indifferent	19	28.4
Total	67	100

Figure 17: Indifference (Participants without multiple switches)

	Count	Share in %
Indifferent	21	53.8
Non-indifferent	18	46.2
Total	39	100

Figure 18: Indifference (Participants with  $4 \leq CE \leq 10$  and satisfying FOSD)

	Count	Share in %
CE inside switch interval	20	46.5
CE outside switch interval	23	53.5
Total	43	100

Figure 19: Indifference (whether CE is inside Machina switching point interval)

We also present the number of observations for specific combinations of CE and S values:

	CE<10	CE=10	CE>10
S<10	14	4	4
S=10	1	1	0
S>10	5	6	9

Figure 20: 2x2 table of CE vs. Switching point

**4.6 What are people choosing at  $X = CE$ ?** Next, we examine what subjects choose when X is their certainty equivalent.

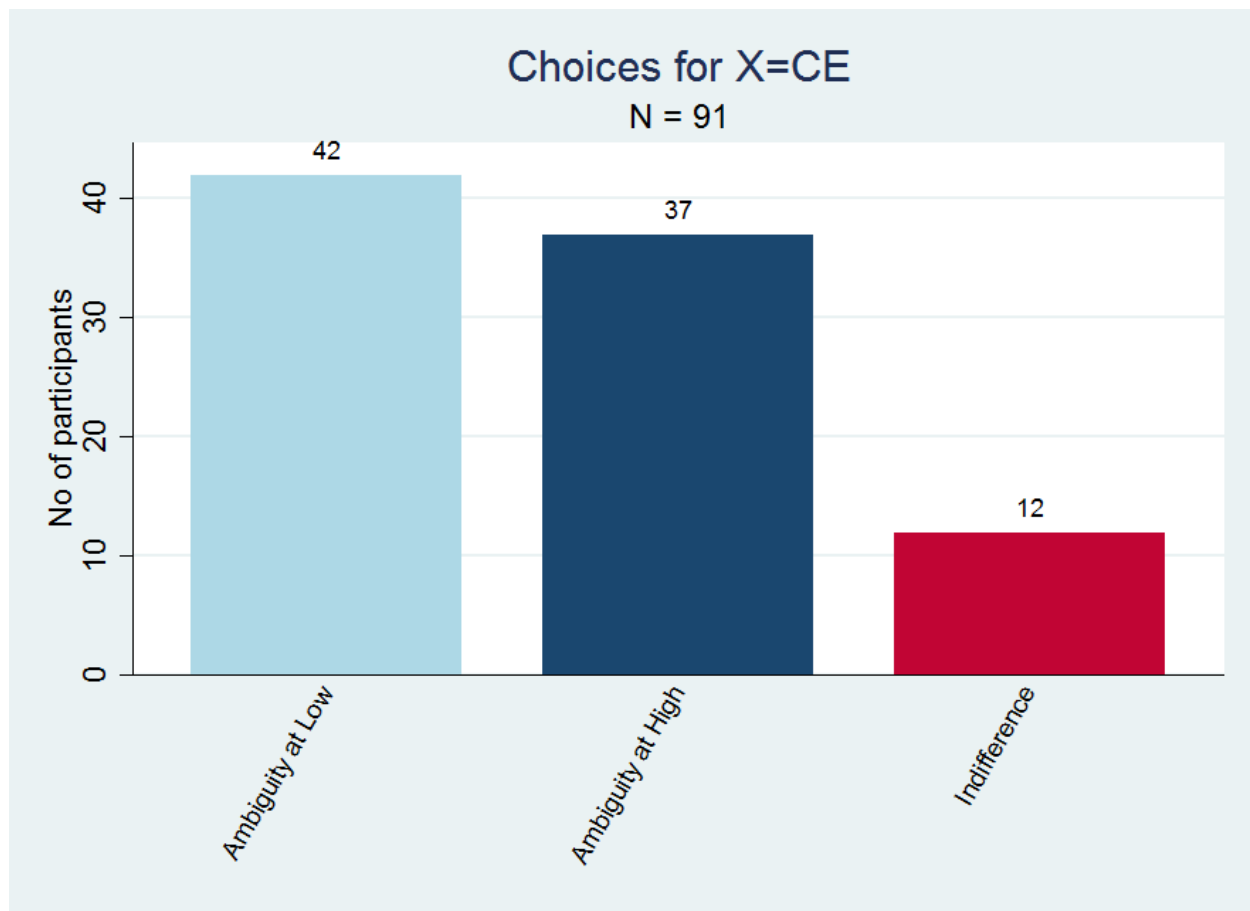


Figure 21: All participants



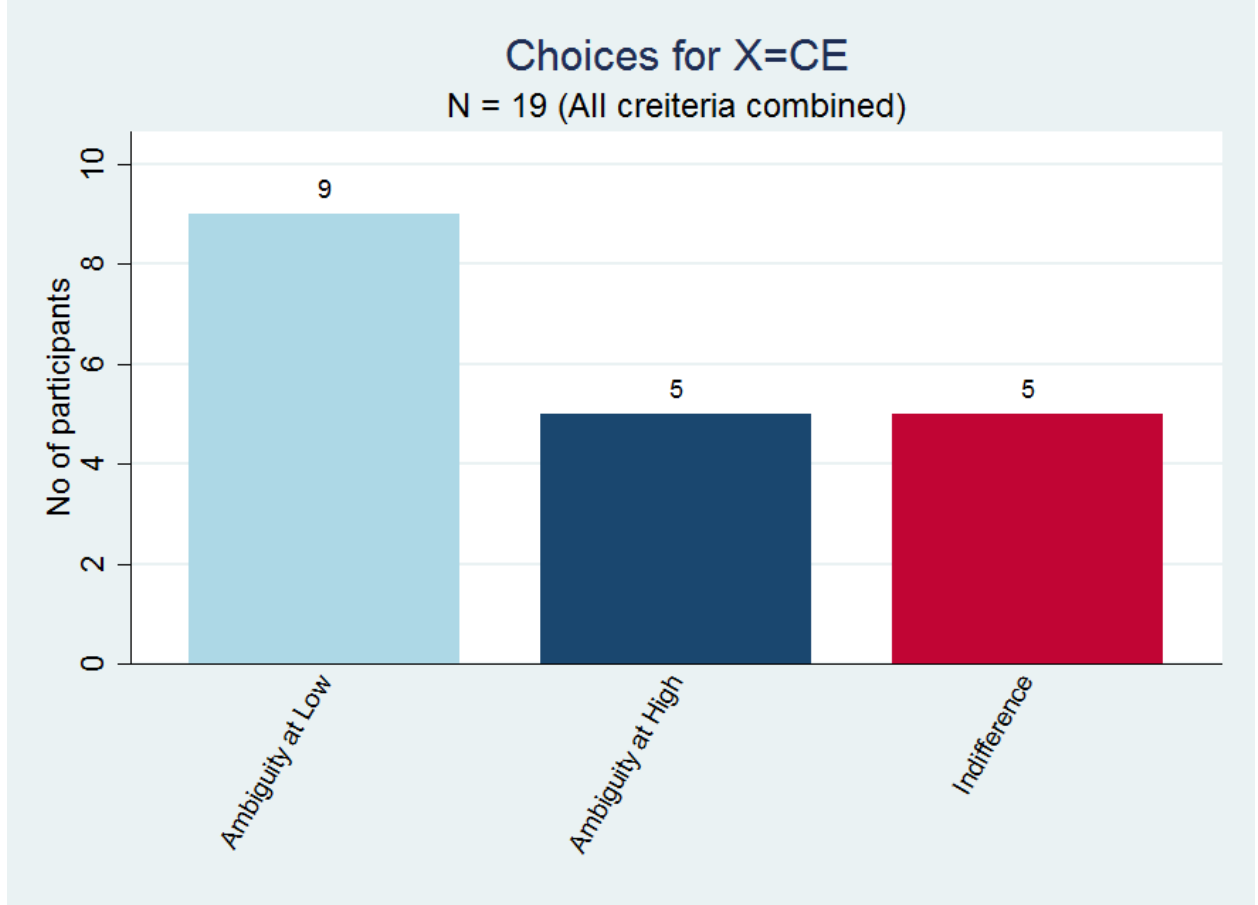


Figure 22: Single-Switchers with  $4 \leq CE \leq 10$  and satisfying FOSD

In both samples, we see that indifference obtains for a minority of subjects. Ambiguity at low outcomes is also somewhat preferred.

**4.7 CE vs. Switching point** To understand the subjects better, we next visualize how separated are their CE and switching points. We plot the CE on the x-axis and the switching point on the y-axis. In each subplot, the 45 degree line is the  $CE = S$  line. This sample includes people who always prefer A or always prefer B (their switching point is represented as 20). Each subplot presents a different sample. Clockwise from the upper left: (i) All participants, (ii)  $CE \in [4, 10]$ , (iii) FOSD, (iv) both. From the number of observations, we can see that the subjects who had  $CE \in [4, 10]$  also satisfied FOSD.

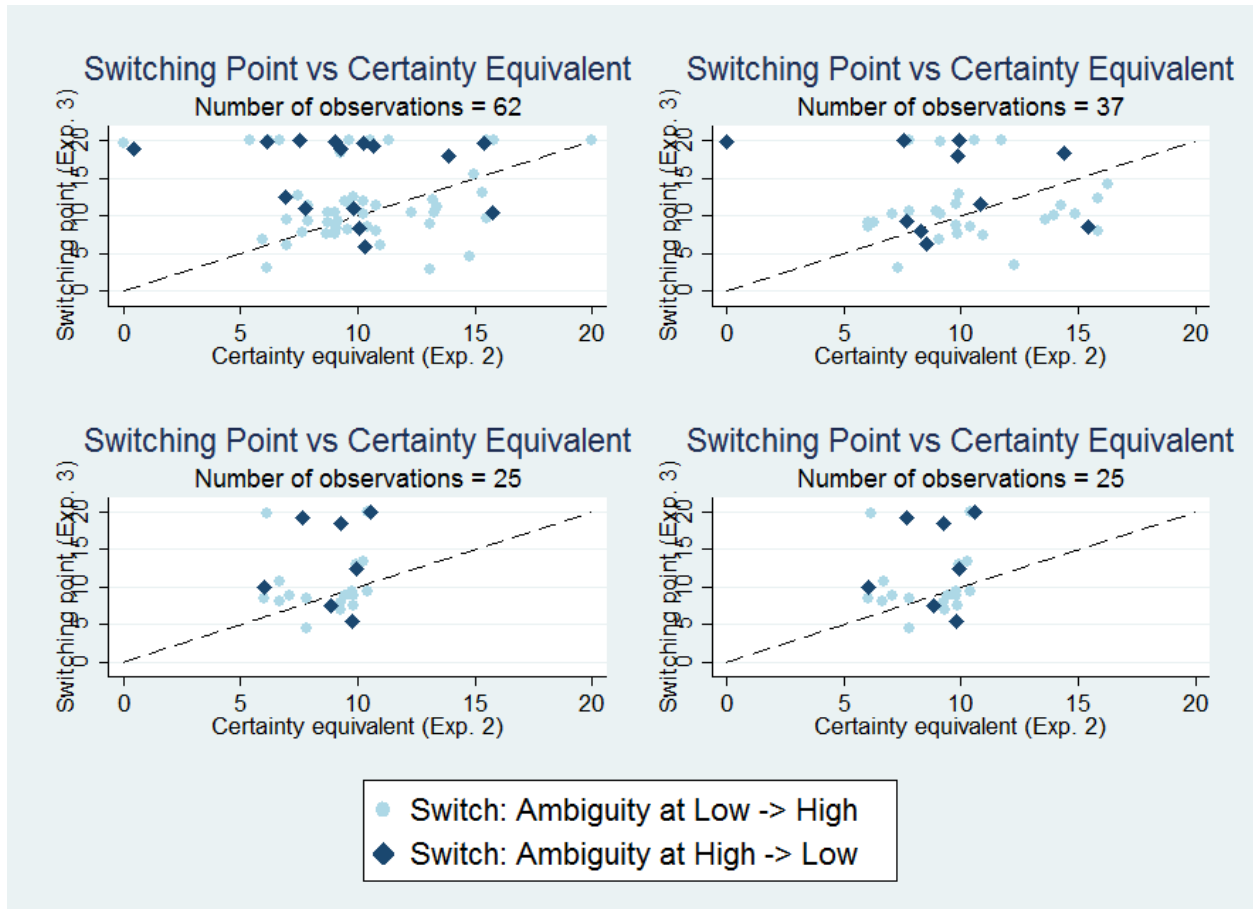


Figure 23: CE vs. Switching point (raw data)

A slightly different visual representation folds the data over the 45 degree line. We fold the data because we do not want to average the responses of some subjects who switch above their CE and other subjects who switch below their CE.

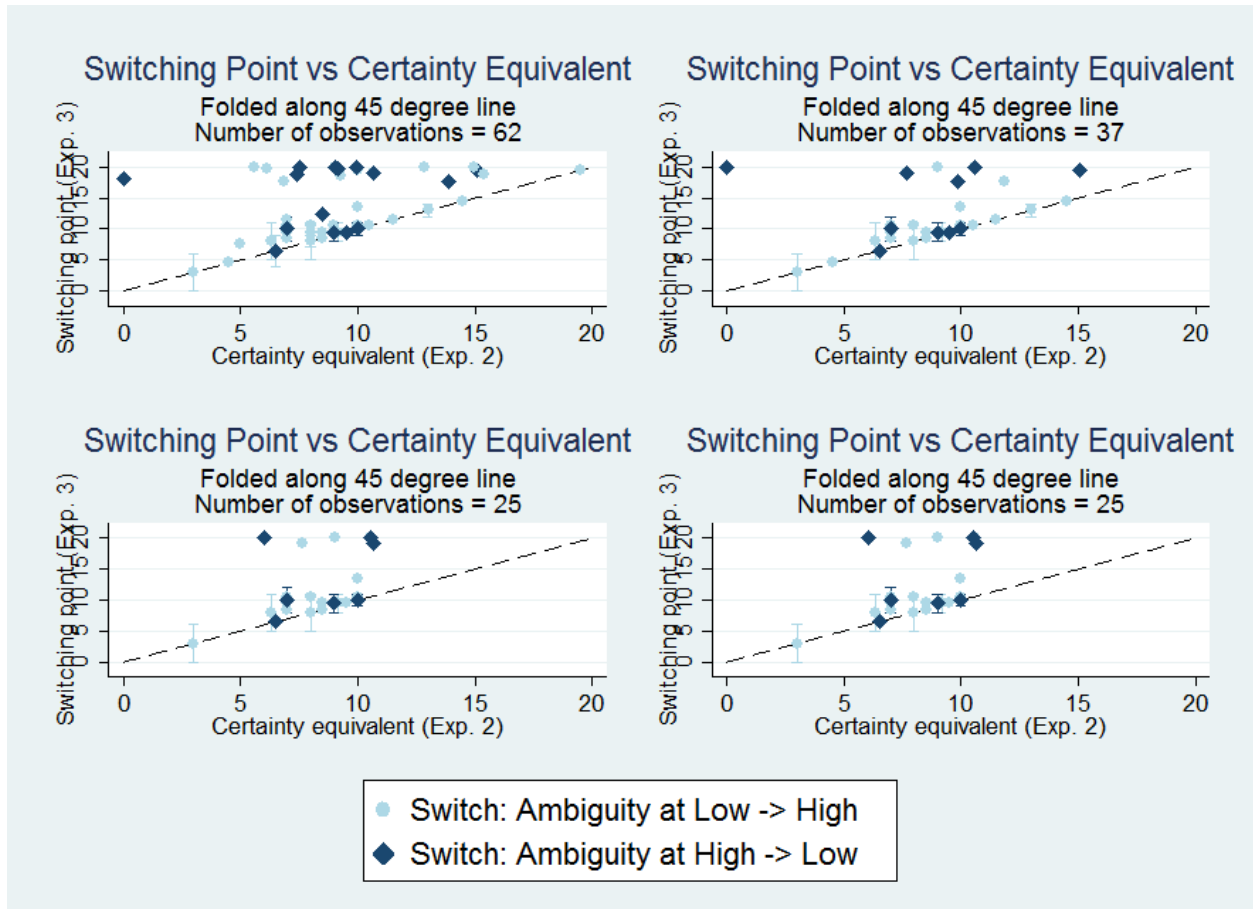


Figure 24: CE vs. Switching point (folded)

This graphical representation corroborates the first result—the null hypothesis of indifference at  $X = CE$  appears to be rejected. Linking this result to the theory would suggest that prominent theories of ambiguity aversion should be rejected.

The final visualization adds a regression line and replaces the some dots with bars when subjects report indifference for a range rather than the data indicating a switching point.

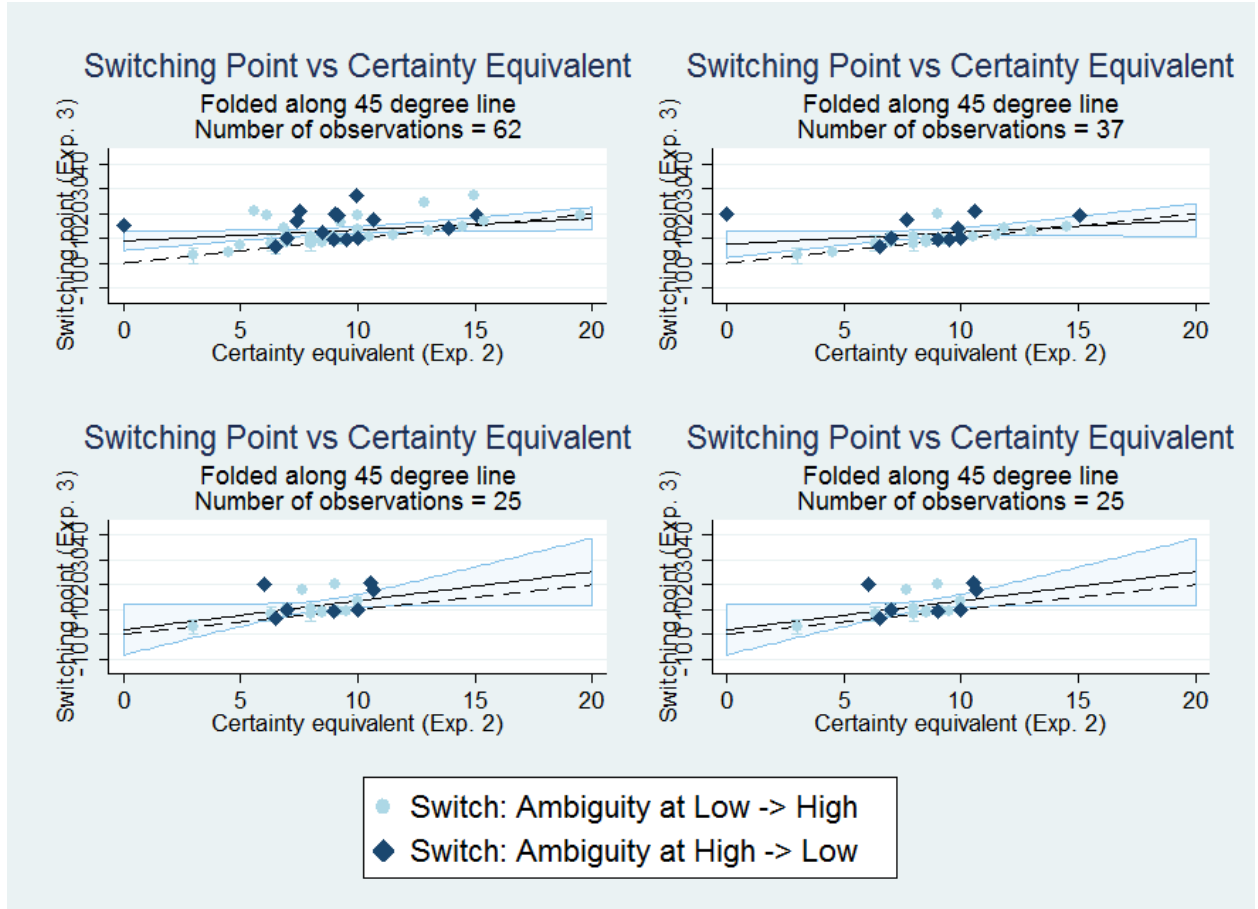


Figure 25: CE vs. Switching point (folded, with regression line)

On this evidence, the confidence interval for the regression line excludes the 45 degree line for the entire set of participants, but in smaller, more ‘rational’ subsamples, the null hypothesis of indifference cannot be rejected.

**4.8 Testing for equality of CE and S** According to models of ambiguity aversion discussed by Machina, subjects should be indifferent when  $X = CE$ . This may mean that they prefer one option below/above their  $CE$  and switch to the other option after  $X$  crosses their  $CE$ . We want to check if this really happens, i.e., whether among subjects who switch, the switch occurs at their  $CE$  (elicited in task 2).

In this test, only subjects with a single switching point are analyzed. This means that if, in the sequence of subjects’ reported choices, A appears first and B appears later, there is no A after the first B (or symmetrically, if B appears first and A appears later, there is no B after the first A). The switching point is calculated as the average value between the position of

last A/B and first B/A. Option “C” (“I am indifferent”) is treated as neutral here: if the last A choice occurs at  $X = 7$  and first B choice at  $X = 11$  and there are “C”s in-between, then  $S$  is calculated to be equal to 9.

All subjects in the experiment have their  $CE$  from task 2, but only some subjects—those who switch just once—have an  $S$ . We only analyze the subsample with subjects with  $S$  available.

#### 4.8.1. Paired T-test

In the paired T-test of equality of means of  $CE$  and  $S$ , we cannot reject the null hypothesis that  $CE$  and  $S$  are equal. This supports the claim that subjects are indifferent at their CE level.

Variable	Obs	Mean	Std Dev
Switching point	44	10.19	3.34
CE	44	10.53	3.16

H0: mean(Switching Point - CE) = 0; p-value for two-sided test: 0.5162

Figure 26: Paired T-test

#### 4.8.2. Sign rank test for matched samples

As in Snedecor and Cochran (1989):

Number of negative comparisons	21
Number of positive comparisons	22
Number of tied comparisons	1
Two-sided p-value	1

Figure 27: Sign rank test

#### 4.8.3. Wilcoxon-Mann-Whitney test (for sample with single switching point)

Wilcoxon-Mann-Whitney test for equality of distribution for  $CE$  and  $S$  (unmatched samples):

Number of observations	44
Expected sum for the first group	1958
Actual sum	1972.5
Z-statistic	.121

Figure 28: Wilcoxon-Mann-Whitney test

#### 4.8.4. Testing whether mean of absolute value of CE and S difference is 0:

T-test for hypothesis that the mean of  $abs(CE - S)$  is 0:

Number of observations	44
t-statistic	7.808
p-value	0

Figure 29: T-test

**4.9 Allais and Machina paradoxes** Next, we present sub-sample analysis, dividing subjects by whether they are Allais consistent or inconsistent. Indifference appears to depend on the answer to Allais (see the questionnaire in the appendix). Among subjects who only switch once, those who are more Allais consistent are more likely to be indifferent.

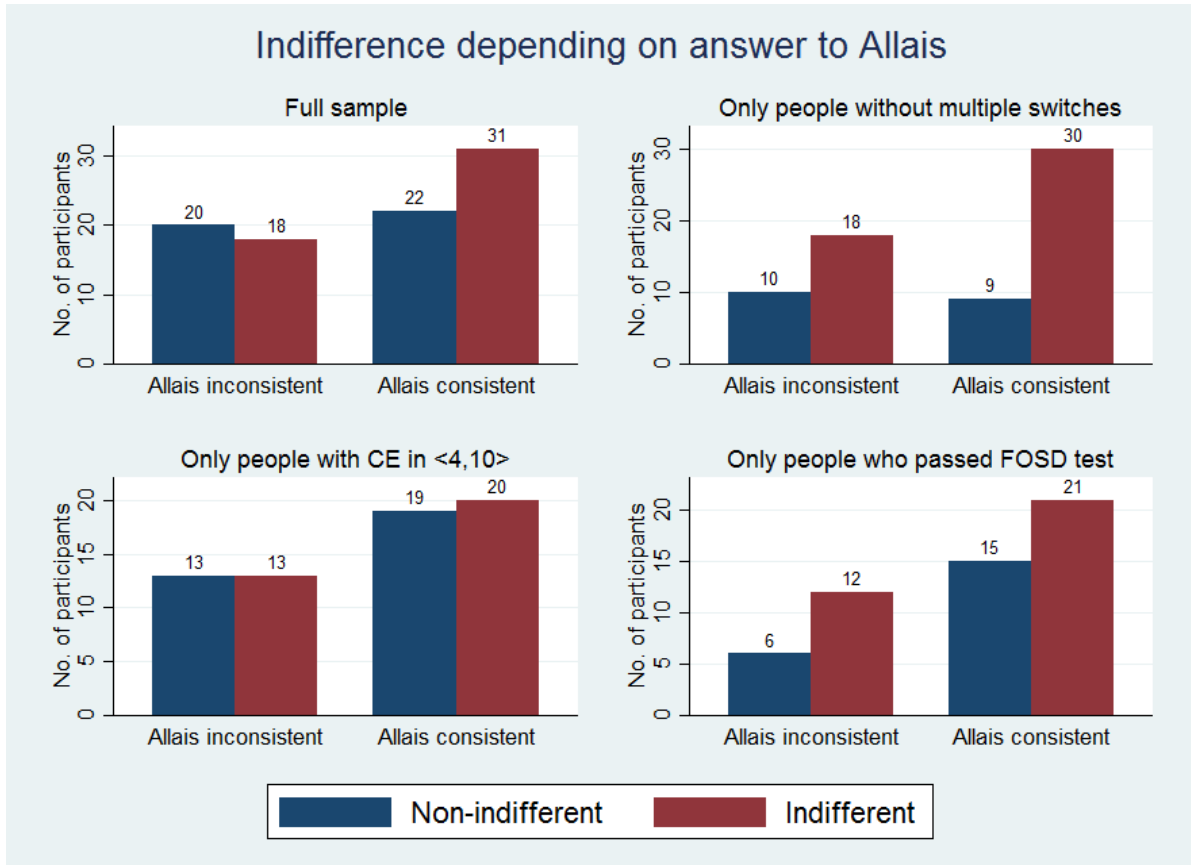


Figure 30: Allais and Machina paradoxes

Next, we treat multiple switchers as indifferent. For single switchers, we treat them the same as in the approach described above: people who switch once are treated as indifferent or non-indifferent.

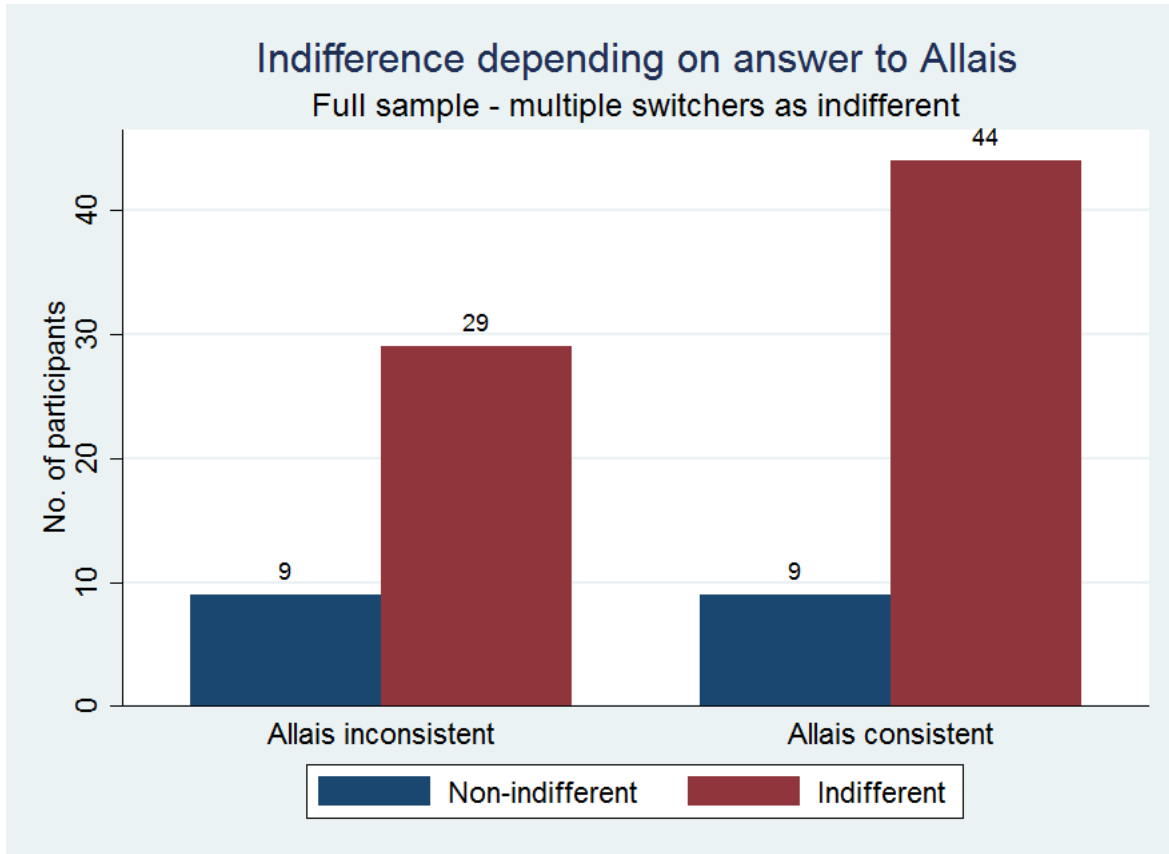


Figure 31: Allais and Machina paradoxes (multiple switches interpreted as indifference)

Now we observe a substantial majority of subjects being indifferent, especially when they are Allais consistent.

**4.10 Does order matter for switch direction?** The order of the lottery presentation was randomized, but we can check if the order influenced the switch direction. We find that the answer is yes, but people still generally switch from Ambiguity at Low to Ambiguity at High.

Fraction of switches from Risk at Low Outcome to Risk at High Outcome depending on the order of options on the answer sheet (normal order lists Risk at High Outcome first).

Group	Obs	Mean	Std Dev
Normal Order	32	.13	.34
Reversed order	11	.18	.4

H0: means are equal; p-value for two-sided test: 0.648

Figure 32: Order and switch direction

The tabulation indicates that the fraction of switches from Ambiguity at High to Ambiguity

at Low depends on the order of options on the answer sheet (normal order lists Ambiguity at Low Outcome first). But even with the reversed order, the majority of subjects switch from Ambiguity at Low to Ambiguity at High.

**4.11 Predictions about direction of switch** Consider Dillenberger and Segal (2013). The value of Acts are computed as the weighted average of values of first-stage lotteries, with weights being subjective probabilities of different states of the world:  $BB, BW, WW$ .

$$W_{Act I} = q_{BB} \cdot V_{Act I}(BB) + q_{BW} V_{Act I}(BW) + q_{WW} V_{Act I}(WW)$$

$$W_{Act II} = q_{BB} \cdot V_{Act II}(BB) + q_{BW} V_{Act II}(BW) + q_{WW} V_{Act II}(WW)$$

Since terms for state  $BW$  are the same for both urns (same payoffs), we may neglect them for comparison purposes. Let's now take Gul's disappointment aversion model with  $\beta$  as the disappointment aversion parameter:

$$\begin{aligned} V_{Act I}(BB) &= \frac{\frac{2}{3}(1+\beta) \cdot 0 + \frac{1}{3} \cdot 100}{1 + \frac{2}{3}\beta} = \frac{100}{3+2\beta} \\ V_{Act I}(WW) &= \frac{\frac{2}{3}(1+\beta) \cdot X + \frac{1}{3} \cdot 100}{1 + \frac{2}{3}\beta} = \frac{100 + 2(1+\beta)X}{3+2\beta} \\ V_{Act II}(BB) &= \frac{\frac{1}{3}(1+\beta) \cdot 0 + \frac{2}{3} \cdot X}{1 + \frac{1}{3}\beta} = \frac{2X}{3+\beta} \\ V_{Act II}(WW) &= \frac{\frac{1}{3}(1+\beta) \cdot 0 + \frac{2}{3} \cdot 100}{1 + \frac{1}{3}\beta} = \frac{200}{3+\beta} \end{aligned}$$

So  $Act I$  is preferred to  $Act II$  if:

$$q_{BB} \frac{100(1+\beta)}{3+2\beta} + q_{WW} \frac{100+2(1+\beta)X}{3+2\beta} > q_{BB} \frac{2X(1+\beta)}{3+\beta} + q_{WW} \frac{200}{3+\beta}$$

For  $q_{WW} = q_{BB}$  (assuming equal probabilities of having two black balls or two white balls)<sup>4</sup>:  $100\beta > 2X\beta$

---

<sup>4</sup>Derivation:

$$\begin{aligned} \frac{100(1+\beta)}{3+2\beta} + \frac{100+2(1+\beta)X}{3+2\beta} &> \frac{2X(1+\beta)}{3+\beta} + \frac{200}{3+\beta} \cdot (3+\beta)(3+2\beta) \\ 2(1+\beta)(3+\beta)100 + 2(1+\beta)(3+\beta)X &> (3+2\beta)2X(1+\beta) + 200(3+2\beta) \\ 600 + 300\beta + 6X(1+\beta) + 200\beta + 100\beta^2 + 2X(1+\beta)\beta &> 600 + 6X(1+\beta) + 400\beta + 4X(1+\beta)\beta \end{aligned}$$



We now divide by  $\beta$ . Let's first assume that  $\beta > 0$ :

$$50 > X$$

So if  $X < 50$ , *Act I* is preferred over *Act II*. Therefore, as  $X$  increases we should observe a switch from *Act I* to *Act II*, which is what we find.

If we now go back and assume that  $\beta < 0$ :

$$50 < X$$

So if  $X > 50$ , *Act I* is preferred over *Act II*. Therefore, as  $X$  increases we should observe a switch from *Act II* to *Act I*.

## 5 Conclusion

The Machina thought experiment is the latest in a series of seminal thought experiments to push the frontiers of both theoretical and empirical research on choice under uncertainty. Machina offers a test of major theories that allow for ambiguity non-neutrality. In Machina's thought experiment, major theories of ambiguity aversion predict indifference, so the thought experiment is posed as a test of these theories. We make two observations. First, probabilistically sophisticated non-Expected Utility (non-EU) decision makers (DM) can fail to be indifferent. We present an example (disappointment aversion) where decision makers have a strict preference. Second, any non-probabilistically sophisticated Expected Utility decision maker is indifferent. We show that for any prior, someone who satisfies the independence axiom will be indifferent. Machina's thought experiment appears at least as much a test of independence as of ambiguity aversion. A challenge with Machina's thought experiment is that it requires knowledge of a subject's certainty equivalent, which we overcome with the PRINCE method. Many theories of ambiguity aversion give a sharp point prediction in Machina's thought experiment. Is the point prediction of indifference about right? Our results—across three experiments—indicate no. We find a strong pattern in which way people shift. This shift is used to support Dillenberger and Segal (2013)'s axiomitization of ambiguity aversion and reject other axiomitizations.

Ambiguity aversion is now used to explain puzzles and promote policies. Financial economists, e.g. Erbas and Mirakhor (2007) and Maenhout (2004), attribute part of the equity premium

to aversion to ambiguity. Health economists interested in targeting public health initiatives could base their policies on correlations found between measures of ambiguity aversion and unhealthy behavior (Sutter et al., 2013). In law, the concept of ambiguity aversion appears to have made the most headway. Ambiguity aversion is argued to result in plea bargaining that is too harsh, as defendants are typically more ambiguity averse than the prosecutor who also faces a repeated situation. The criminal process therefore is systematically affected by asymmetric ambiguity aversion, which the prosecution can exploit by forcing defendants into harsh plea bargains, as Segal and Stein (2005) contend. Uncertain risks surrounding environmental protection and medical malpractice have led to calls to provide more scientific data to ameliorate the relevance of ambiguity aversion in individuals' policy preferences, e.g., by Viscusi and Zeckhauser (2006) and Farber (2010). And the theory and practice of statutory interpretation is rife with ambiguity, as Farnsworth et al. (2010) argue. Ambiguity aversion has also been applied to contracts (Talley, 2009) and tax compliance (Lawsky, 2013). Yet little is known at the present time about ambiguity non-neutrality. Conventional wisdom now seems to be that people are ambiguity-neutral or -averse, but even that statement cannot be defended any more, as Halevy (2007) finds that in his sample half of the subjects are ambiguity-averse, but a remarkable 35% ambiguity-seeking. Thus, more research is needed to find out more about ambiguity attitudes.

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## A Instructions

## EXPERIMENT 1

Participant number: \_\_

Please now enter your participant number in the space above.

In this first of 3 experiments you will decide between two options. In both options your payoff depends on the result of a random draw from a drum filled with balls and some unknown amount CHF X.

On the table in the middle of the room there is a drum. We will conduct 2 draws. First we will fill the drum as shown in the table "Option A" below and randomly draw a single ball. Then we will fill the drum as shown in the table for "Option B" below, and randomly draw a single ball.

When asked by the experimenter to do so, draw one sealed white envelope. Each participant will draw a white envelope. Each of these envelopes contains a note. The notes are identical except for the random value X which differs. The CHF X is a random number between CHF 0,00 and CHF 20,00. All possible numbers have equal probability.

The value of X is printed on the note inside the sealed envelope. You will only learn X once experiment 1 is over. Thus DO NOT OPEN YOUR ENVELOPE. It will be opened later by an experimenter in your presence. If you open your envelope yourself, you will not get paid for this experiment.

Here is what the note in each of the envelopes looks like:

Option A		Option B	
Balls in drum	Money you get when a ball of this color is drawn	Balls in drum	Money you get when a ball of this color is drawn
4 red balls	CHF X	2 red balls	CHF X
3 white balls	CHF 9	3 white balls	CHF 9
3 black balls	CHF 7	5 black balls	CHF 7

You can choose whether you want to get Option A or Option B. But since your envelope may contain any value of X between 0,00 and 20,00 please give us general instructions whether you want Option A or Option B depending on the value of X.

Do so by selecting one of the following options and indicating a threshold:

- ☐ If X is below or equal to CHF \_\_ \_\_ then I want **Option A**, else I will receive **Option B**.
- ☐ If X is below or equal to CHF \_\_ \_\_ then I want **Option B**, else I will receive **Option A**.

*Please give this sheet to the experimenter when asked to do so. He will later return it to you, and in the end you have to hand it to the cashier to get paid.*

## EXPERIMENT 2

Participant number: \_ \_

Please now enter your participant number in the space above.

In this experiment you will decide whether you prefer to play a lottery, in which the payoff is dependent on the color of a ball randomly drawn from a drum, or to receive a guaranteed amount of money.

When asked by the experimenter to do so, draw one sealed green envelope. Each participant will draw a green envelope. Each of these envelopes contains a note. The notes are identical except for the random value  $X$  which differs. The CHF  $X$  is a random number between CHF 0,00 and CHF 20,00. All possible numbers have equal probability.

The value of  $X$  is printed on the note inside the sealed envelope. You will only learn  $X$  once experiment 2 is over. Thus **DO NOT OPEN YOUR ENVELOPE**. It will be opened later by an experimenter in your presence. If you open your envelope yourself, you will not get paid for this experiment.

In this experiment 2 the note has two options: participating in the lottery or receiving a guaranteed amount of money. The guaranteed amount of money is CHF  $X$ .

Here is what the note in each of the envelopes looks like:

The drum is filled with 20 balls, 10 of which are white and 10 of which are black. The experimenter will draw one ball from the drum at random.

**Option A**

If the ball is black you get CHF 20. If the ball is white you get nothing.

**Option B**

Regardless of what color is drawn, you get CHF  $X$ .

Since your envelope may contain any value of  $X$  between 0,00 and 20,00 please give us general instructions whether you want Option A or Option B. Do so by specifying a threshold:

IF  $X$  is below or equal to CHF  I want Option A, otherwise I will receive Option B.

*Please give this sheet to the experimenter when asked to do so. He will later return it to you, and in the end you have to hand it to the cashier to get paid.*



### EXPERIMENT 3

Participant number: \_\_

Please now enter your participant number in the space above.

In experiment 3 you will choose one of two options. In both options your payoff depends on the result of a random draw from a drum filled with balls and some unknown amount CHF X.

On the table in the middle of the room there is a drum. We will conduct a single draw. The experimenter will show you the drum with 20 red balls already in it. He will also show you a box with 40 white balls, and a box with 40 black balls. Later, in secret he will add exactly 40 white and black balls to the drum in addition to the 20 red ones already in the drum. The additional 40 balls may be any combination of black and white balls. The experimenter could for example add 7 black balls and 33 white balls or just add 40 white balls, or he could add 12 white balls and 28 black balls, or.... He can do whatever he likes as long as the sum of white and black balls in the drum is exactly 40, and the number of red balls in the drum remains at 20.

When asked by the experimenter to do so, draw one sealed blue envelope. Each participant will draw a blue envelope. Each of these envelopes contains a note. The notes are identical except for the random value X which differs. The CHF X is a random number between CHF 0 and CHF 20. In this experiment these numbers will be only whole Swiss Francs, so CHF 0, CHF 1, CHF 2,....., CHF 19, CHF 20. Thus there are 21 possible numbers, and they have equal probability.

The value of X is printed on the note inside the sealed envelope. You will only learn X once experiment 3 is over. Thus DO NOT OPEN YOUR ENVELOPE. It will be opened later by an experimenter in your presence. If you open your envelope yourself, you will not get paid for this experiment.

Here is what the note in each of the envelopes looks like:

The ball will be drawn from a drum with 20 red balls and 40 balls which may be any combination of white and black balls. You do not know exactly how many white/black balls are in the drum. There are 60 balls in total in the drum. You can choose one of two options. The option payoff is dependent on the color of the ball drawn from the drum:			
	20 balls	40 balls	
	Red ball drawn	Black ball drawn	White ball drawn
Option A	CHF 20	CHF 0	CHF X
Option B	CHF 0	CHF X	CHF 20

PLEASE NOW ANSWER THE QUESTIONS ON THE OTHER SIDE OF THIS PAPER



Please give us instructions, for each possible value of X that your envelope may contain, whether you want Option A or B. Do so by ticking the option you prefer for every possible value of X (so put exactly one tick in each of the 21 rows). If you are indifferent, you will get the payoff from Option A or Option B – it will be randomly determined which one.

If X is....	...I want Option A	...I want Option B	I am indifferent
CHF 0			
CHF 1			
CHF 2			
CHF 3			
CHF 4			
CHF 5			
CHF 6			
CHF 7			
CHF 8			
CHF 9			
CHF 10			
CHF 11			
CHF 12			
CHF 13			
CHF 14			
CHF 15			
CHF 16			
CHF 17			
CHF 18			
CHF 19			
CHF 20			

Note: This question does not have a "correct" answer. So just think row by row which option you feel is better.

*Please give this sheet to the experimenter when asked to do so.  
He will later return it to you, and in the end you have to hand it to the cashier to get paid.*

Appendix Figure A.4: Machina Task (page 2)

## QUESTIONNAIRE      Participant number:\_\_\_\_\_

Please now enter your participant number in the space above

Please answer the following questions.

### QUESTIONNAIRE PART 1

A doctor gives you 3 pills, and tell you to take 1 pill every 30 minutes starting right away. After how many minutes will you run out of pills? \_\_\_\_\_minutes

A meal, including a beverage costs CHF 12 in total. The food costs 5 times as much as the beverage. How much does the food cost? \_\_\_\_\_

A population of a town halves every month due to a plague. 1 000 people are still alive after 10 months. After how many months were 2 000 people alive? \_\_\_\_\_

### QUESTIONNAIRE PART 2

In this part of the questionnaire we will ask you to make a choice between pairs of lotteries. These lotteries will NOT be paid out. Please answer as you think you would if the choice were real rather than hypothetical. Note that in neither of these questions there is a unique correct answer.

QUESTION: Suppose you got offered a choice between these 2 lotteries. Suppose you would not have to pay anything for either of them, and you could choose exactly one lottery. Which one would you choose?

O Lottery A:    CHF 1 Million for sure

O Lottery B:    1% Chance of Nothing.  
                      89% Chance of CHF 1 Million.  
                      10% Chance of CHF 5 Million

QUESTION: Now imagine you did not get the choice offered above, but instead got offered a choice between the 2 lotteries below for free. Suppose you got offered a choice between these 2 lotteries for free. And you could choose exactly one.  
Which one would you choose?

O Lottery C: 89% Chance of Nothing  
11% Chance of 1 Million CHF

O Lottery D: 90% Chance of Nothing.  
10% Chance of 5 Million CHF

### QUESTIONNAIRE PART 3

1. What is your country of citizenship: \_\_\_\_\_

2. What is your mother tongue (native language): \_\_\_\_\_

3. What is your age? \_\_\_\_\_ years

4. What is your gender? ☐ Male ☐ Female

5. In what kind of program are you currently enrolled?

☐ Bachelor's program

☐ Master's program

☐ Not a student

6. In which year do you think you will graduate from your current program?

☐ 2014

☐ 2015

☐ 2016

☐ 2017

☐ 2018

☐ Later

☐ I am not a student.

7. What is your field of study? \_\_\_\_\_

8. Have many times have you participated in experiments before today (in this ETH laboratory or at the University of Zurich) ?

☐ Never

☐ Once

☐ 2times

☐ 3 times

☐ \_\_\_\_\_

9. How hard to understand were today's experiments? For each experiment choose the most appropriate option. Please note that this will not influence your payoff and will not be linked to your personal data.

	I didn't understand the instructions	I understood the instructions, but I didn't know what answer to give	Everything was clear	Other (provide extra details)
Experiment 1				
Experiment 2				
Experiment 3				

Appendix Figure A.6: Questionnaire (page 2)